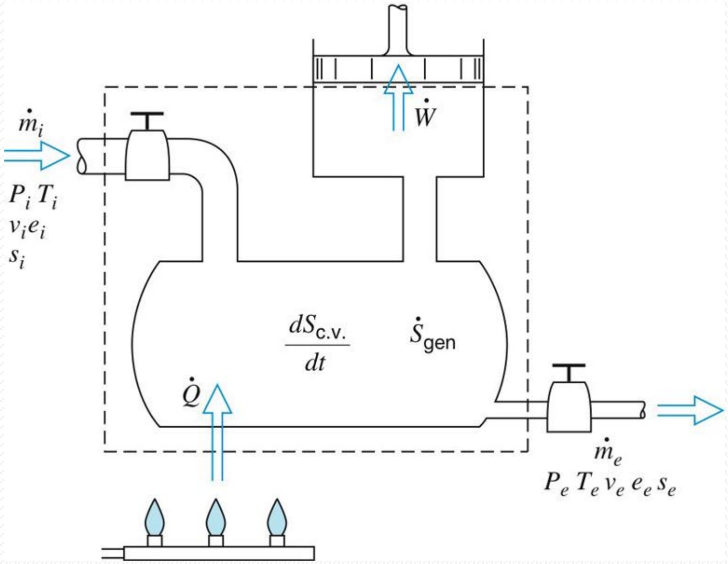
Chapter 9 Entropy Equation for a Control Volume

9.1. The Second Law of Thermodynamics for a Control Volume

We can drive the application formula of the second law of thermodynamics for a control volume, by starting the second law for a control mass

The second law expressed as a change of the entropy for a control mass in a rate form from



$$\frac{dS_{cm}}{dt} = \sum \frac{\dot{Q}}{T} + \dot{S}_{gen}$$

Add the contributions from the mass flow rates in and out of the control volume

$$\dot{S}_{cv} = \overbrace{\sum \dot{m}_i s_i - \sum \dot{m}_e s_e} + \sum \frac{\dot{Q}_{cv}}{T} + \dot{S}_{gen}$$

rate of change = + in – out + generation

The balance of entropy as an equation then states that the rate of change in total entropy inside the control volume is equal to the net sum of fluxes across the control surface plus the generation rate.

These fluxes are mass flow rates carrying a level of entropy and the rate of heat transfer that takes place at a certain temperature.

$$\dot{S}_{cv} = \sum \dot{m}_i\, s_i - \sum \dot{m}_e\, s_e + \sum \frac{\dot{Q}_{cv}}{T} + \dot{S}_{gen}$$

$$\boxed{S_{cv} = \int \rho\, s\, dV = m_{cv}\, s = m_A\, s_A + m_B\, s_B + m_C\, s_C + \ldots}$$

$$\boxed{\dot{S}_{gen} = \int \rho\, \dot{s}_{gen}\, dV = \dot{S}_{gen\,A} + \dot{S}_{gen\,B} + \dot{S}_{gen\,C} + \ldots}$$

The accumulation and generation terms cover the total control volume

$$\boxed{\sum \frac{\dot{Q}_{cv}}{T} = \int \frac{d\dot{Q}}{T} = \int_{surface} \frac{(\dot{Q}/A)}{T}\, dA}$$

and the heat transfer over the control surface

$$\dot{S}_{cv} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{cv}}{T} + \dot{S}_{gen}$$

The generation term in above equation is necessarily positive (or zero), such that an inequality is often written as

$$\dot{S}_{cv} \geq \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{cv}}{T}$$

the equality applies to internally reversible processes and the inequality to internally irreversible processes.

If there is no mass flow into or out of the control volume, it simplifies to a control mass

9.2 The Steady–State Process and the Transient Process

Steady-State Process

There is no change with time of the entropy per unit mass at any point within the control volume, and therefore the rate of accumulation covering the total control volume equals to zero.

$$\dot{S}_{cv} = 0$$

so that, for the steady–state process

$$\sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum_{cv} \frac{\dot{Q}_{cv}}{T} + \dot{S}_{gen}$$

If in a steady-state process there is only one inlet and only one exit, we can write

$$\dot{m}(s_e - s_i) = \sum_{cv} \frac{\dot{Q}_{cv}}{T} + \dot{S}_{gen}$$

and dividing the mass flow rate out gives

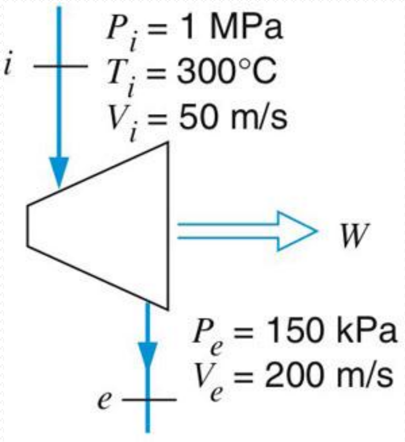
$$(s_e - s_i) = \sum_{cv} \frac{q}{T} + s_{gen}$$

S_{gen} is always greater than or equal to zero, for an adiabatic process it follows that

$$s_e = s_i + s_{gen} \geq 0$$

where the equality holds for a reversible adiabatic process

EXAMPLE 9.1 Steam enters a steam turbine at a pressure of 1 MPa, a temperature of 300°C, and a velocity of 50 m/s. The steam leaves the turbine at a pressure of 150 kPa and a velocity of 200 m/s. Determine the work per kilogram of steam flowing through the turbine, assuming the process to be reversible and adiabatic.



From the steam tables
The initial state

h_i = 3052.1 kJ/kg
s_i = 7.1251 kJ/kg.K

The continuity equation $\dot{m}_e = \dot{m}_i = \dot{m}$

the first law $h_i + \frac{V_i^2}{2} = h_e + \frac{V_e^2}{2} + w$

the second law $s_e = s_i$

the final state
P_e = 0.15 Mpa
$s_e = s_i$ = 7.1251 kJ/kg.K

The quality and enthalpy of the steam leaving the turbine can be determined

$$s_e = 7.1251 = s_f + x_e s_{fg} = 1.4335 + x_e(7.2234 - 1.4336) \qquad x_e = 0.9830$$

$$h_e = h_f + x_e h_{fg} = 467.13 + 0.983 * (2693.4 - 467.13) = 2655.55$$

$$w = 3052.1 + \frac{50^2}{2*1000} - 2655.55 - \frac{200^2}{2*1000} = 377.8\, kJ/kg$$

EXAMPLE 9.3 An inventor reports having a refrigeration compressor that receives saturated Refrigerant-134a vapor at -20°C and delivers the vapor at 1 MPa and 40°C. The compression process is adiabatic. Does the process described violate the second law?

Because this is a steady-state adiabatic process, we can write the second law as

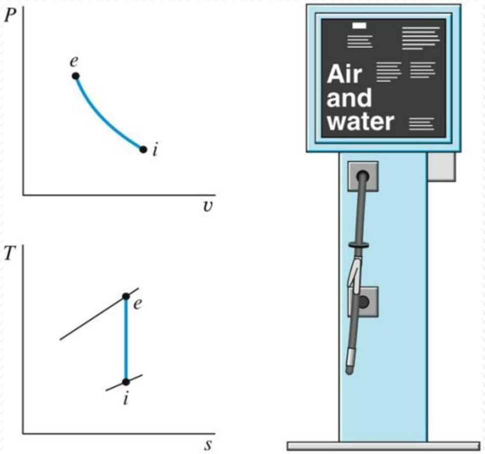
$$s_e \geq s_i$$

From the R-134a tables
$s_e = 1.7157$ kJ/kg K,
$s_i = 1.7432$ kJ/kg K

Therefore, $s_e < s_i$, whereas for this process the second law requires that $s_e \geq s_i$

The process described involves a violation of the second law and thus would be impossible.

EXAMPLE 9.4 An air compressor in a gas station, see Figure, takes in a flow of ambient air at 100 kPa, 290 K, and compresses it to 1000 kPa in a reversible adiabatic process. We want to know the specific work required and the exit air temperature.



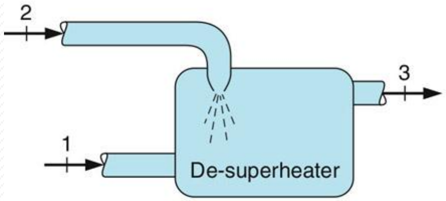
Continuity	$\dot{m}_e = \dot{m}_i = \dot{m}$
Energy	$\dot{m}\, h_i = \dot{m}\, h_e + \dot{W}_c$
Entropy	$\dot{m}\, s_i + \dot{S}_{gen} = \dot{m}\, s_e$
Process	Reversible $\dot{S}_{gen} = 0$

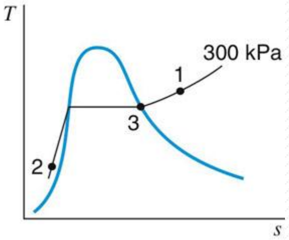
Use constant specific heat from Table 2.2 $C_P = 1.0035$ kJ/kg.K, $k = 1.4$. Entropy equation gives constant s, which gives the relation

$$T_e = T_i \left(\frac{P_e}{P_i} \right)^{(k-1)/k} = 290 \left(\frac{1000}{100} \right)^{(1.4-1)/1.4} = 559.9\, K$$

$$w = h_i - h_e = C_P\,(T_i - T_e) = 1.0035 * (290 - 559.9) = -271 kJ/kg$$

EXAMPLE 9.5 A de-superheater works by injecting liquid water into a flow of superheated steam. With 2 kg/s at 300 kPa, 200°C, steam flowing in, what mass flow rate of liquid water at 20°C should be added to generate saturated vapor at 300 kPa? We also want to know the rate of entropy generation in the process.





no external heat transfer, and no work

Continuity $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$

Entropy $\dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{S}_{gen} = \dot{m}_3 s_3$

Process P = constant, $\dot{W} = 0$, and $\dot{Q} = 0$

h_1 = 2865.45 kJ/kg s_1 = 7.3390 kJ/kg.K

h_2 = 84.1 kJ/kg s_2 = 0.29625 kJ/kg.K

h_3 = 2724.7 kJ/kg s_3 = 6.9909 kJ/kg.K

$$\dot{m}_2 = \dot{m}_1 \frac{h_1 - h_3}{h_3 - h_2} = 2\frac{2865.45 - 2724.7}{2724.7 - 84.1} = 0.1066\, kg/s \quad \dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 2.1066\ kg/s$$

$$\dot{S}_{gen} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 \quad \dot{S}_{gen} = 2.1066 * 6.9909 - 2 * 7.3390 - 0.1066 * 0.29625$$

$$\dot{S}_{gen} = 0.0176\ kW/K$$

Transient Process

For the transient process, the second law for a control volume

$$\frac{d(ms)_{cv}}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{cv}}{T} + \dot{S}_{gen}$$

$$\int_0^t \frac{d(ms)_{cv}}{dt} dt = (m_2 s_2 - m_1 s_1)_{cv}$$

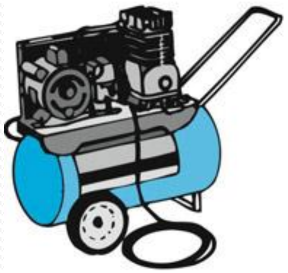
$$\sum \int_0^t \dot{m}_i s_i \, dt = \sum m_i s_i, \qquad \sum \int_0^t \dot{m}_e s_e \, dt = \sum m_e s_e$$

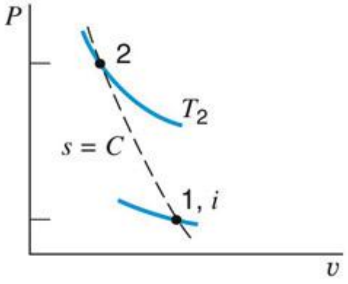
$$\int_0^t \dot{S}_{gen} \, dt = {}_1S_{2\,gen}$$

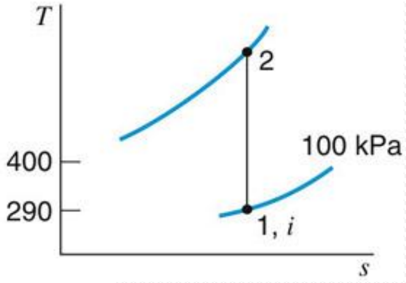
the second law for the transient process

$$\boxed{(m_2 s_2 - m_1 s_1)_{cv} = \sum m_i s_i - \sum m_e s_e + \int_0^t \sum \frac{\dot{Q}_{cv}}{T} dt + {}_1S_{2\,gen}}$$

EXAMPLE 9.6 Assume an air tank has 40 L of 100 kPa air at ambient temperature 17°C. The adiabatic and reversible compressor is started so that it charges the tank up to a pressure of 1000 kPa and then it shuts off. We want to know how hot the air in the tank gets and the total amount of work required to fill the tank.







Continuity $\dot{m}_2 - \dot{m}_1 = \dot{m}_i$

Energy $m_2 u_2 - m_1 u_1 = Q_{12} - W_{12} + m_i h_i$

Entropy $m_2 s_2 - m_1 s_1 = \int \delta Q / T + {}_1S_{gen2} + m_i s_i$

Process Adiabatic ${}_1Q_2 = 0$,
Reversible ${}_1S_{2\,gen} = 0$, $s_1 = s_i$

$$m_2 s_2 = m_1 s_1 + m_i s_i = (m_1 + m_i) s_1 = m_2 s_1 \Rightarrow s_2 = s_1$$

$$s^o_{T2} = s^o_{T1} + R \ln \frac{P_2}{P_1} = 2.0318 + 0.287 * \ln 10 = 2.6926\, kJ / kg.K$$

Interpolate in Table 2.4.3
T_2=554.9 K, u_2=402.97 kJ/kg

$$m_1 = P_1 V_1 / RT_1 = 100 * 0.04 / (0.287 * 290) = 0.04806\, kg$$

$$m_2 = P_2 V_2 / RT_2 = 1000 * 0.04 / (0.287 * 554.9) = 0.2512\, kg \quad \dot{m}_i = \dot{m}_2 - \dot{m}_1 = 0.20314\, kg$$

$${}_1W_2 = m_i h_i + m_1 u_1 - m_2 u_2 \quad {}_1W_2 = 0.20314 * 291.83 + 0.04806 * 208.60 - 0.2512 * 402.97$$

$${}_1W_2 = -31.93\, kJ$$

9.3 The Reversible Steady–State Process

An expression can be derived for the work in a reversible, adiabatic, steady-state process

A steady-state process involves a single flow of fluid into and out of the control volume,

the first law, $$q + h_i + \frac{V_i^2}{2} + g\,Z_i = h_e + \frac{V_e^2}{2} + g\,Z_e + w$$

the second law $$s_i + s_{gen} + \int \frac{\delta Q}{T} = s_e$$

$$\delta s_{gen} + \delta q / T = ds \qquad \Rightarrow \qquad \delta q = Tds - T\delta s_{gen}$$

$$\delta q = Tds - T\delta s_{gen} = dh - v\,dP - T\delta s_{gen}$$

$$q = \int_i^e \delta q = \int_i^e dh - \int_i^e v\,dP - \int_i^e T\delta s_{gen}$$

$$= h_e - h_i - \int_i^e v\,dP - \int_i^e T\delta s_{gen}$$

$$w = q + h_i - h_e + \frac{1}{2}(V_i^2 - V_e^2) + g\,(Z_i - Z_e)$$

$$= h_e - h_i - \int_i^e v\,dP - \int_i^e T\delta s_{gen} + h_i - h_e + \frac{1}{2}(V_i^2 - V_e^2) + g\,(Z_i - Z_e)$$

$$w = -\int_i^e v\,dP + \frac{1}{2}(V_i^2 - V_e^2) + g\,(Z_i - Z_e) - \int_i^e T\delta s_{gen}$$

A simplified version of last equation arises when we consider a reversible flow of an incompressible fluid (v = constant). The first integral is then readily done to give

$$w = -v(P_e - P_i) + \frac{V_i^2 - V_e^2}{2} + g(Z_i - Z_e)$$

which is called the extended Bernoulli equation

$$vP_i + \frac{V_i^2}{2} + gZ_i = vP_e + \frac{V_e^2}{2} + gZ_e$$

For the steady-state process with no change in kinetic and potential energies, we have the relations

$$w = -\int_1^2 v\,dP \qquad and \qquad Pv^n = constant$$

$$w = -\int_1^2 v\,dP = -C\int_1^2 P^{-1/n}\,dP$$

$$w = -\frac{n}{n-1}(P_e v_e - P_i v_i) = -\frac{nR}{n-1}(T_e - T_i)$$

If the process is isothermal, then n = 1 and the integral becomes

$$w = -\int_1^2 v\,dP = -C\int_1^2 \frac{dP}{P} = -P_i v_i \ln\frac{P_e}{P_i}$$

EXAMPLE 9.7 Calculate the work per kilogram to pump water isentropically from 100 kPa and 30°C to 5 MPa.

Since the process is steady, state, reversible, and adiabatic, and because changes in kinetic and potential energies can be neglected, we have

First law : $w = h_i - h_e$

Second law: $s_e = s_i$

From the steam tables, $v_i = 0.001\,00475$ m³/kg. Assuming that the specific volume remains constant

$$-w = \int_1^2 v\,dP = v(P_2 - P_1) = 0.00100475 * (5000 - 100)$$

$$-w = 4.92\,kJ/kg$$

Example 9.8 Consider a nozzle used to spray liquid water. If the line pressure is 300 kPa and the water temperature is 20°C, how high a velocity can an ideal nozzle generate in the exit flow?

For this single steady-state flow, we have no work or heat transfer, and since it is incompressible and reversible, the Bernoulli equation applies, giving

$$v P_i + \frac{V_i^2}{2} + g Z_i = v P_i + 0 + 0 = v P_e + \frac{V_e^2}{2} + g Z_e = v P_o + \frac{V_e^2}{2} + 0$$

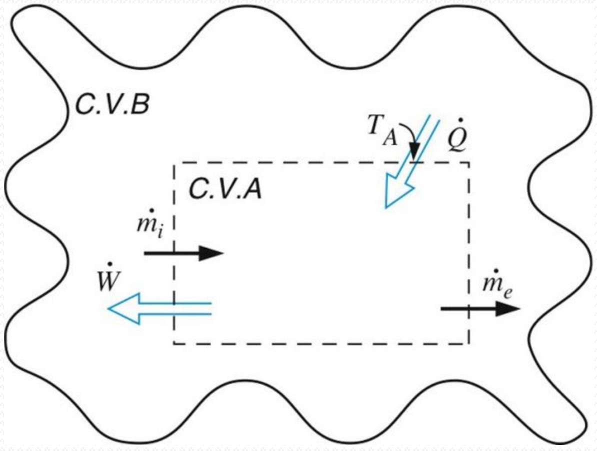
and the kinetic energy becomes $\frac{1}{2} V_e^2 = v(P_i - P_o)$

$v = v_f = 0.001002\ m^3/kg$ at 20°C from the steam tables

$$V_e = \sqrt{2v(P_i - P_o)} = \sqrt{2 * 0.001002 * (300 - 100) * 1000}$$

$$= 20\, m/s$$

9.4 Principle of the Increase of Entropy



the entropy balance equation for the control volumes

$$\dot{S}_{cv\,A} = \dot{m}_i\, s_i - \dot{m}_e\, s_e + \frac{\dot{Q}}{T_A} + \dot{S}_{gen\,A}$$

$$\dot{S}_{cv\,B} = -\dot{m}_i\, s_i + \dot{m}_e\, s_e - \frac{\dot{Q}}{T_B} + \dot{S}_{gen\,B}$$

the net rate of change of S for the total world

$$\dot{S}_{net} = \dot{S}_{cv\,A} + \dot{S}_{cv\,B} = \dot{m}_i\, s_i - \dot{m}_e\, s_e + \frac{\dot{Q}}{T_A} + \dot{S}_{gen\,A}$$

$$-\dot{m}_i\, s_i + \dot{m}_e\, s_e - \frac{\dot{Q}}{T_A} + \dot{S}_{gen\,B}$$

$$= \dot{S}_{gen\,A} + \dot{S}_{gen\,B} \geq 0$$

$$\Delta S_{net} = \Delta S_{CV\,A} + \Delta S_{surr\,B}$$

in which the control volume A term is

$$\Delta S_{CV\,A} = (m_2 s_2 - m_1 s_1)_{CV\,A}$$

$$\Delta S_{surr\,B} = m_e\, s_e - m_i\, s_i - \frac{Q}{T_B}$$

Example 9.9 Saturated vapor R-410a enters the uninsulated compressor of a home central air-conditioning system at 5◦C. The flow rate of refrigerant through the compressor is 0.08 kg/s, and the electrical power input is 3 kW. The exit state is 65◦C, 3000 kPa. Any heat transfer from the compressor is with the ambient at 30◦C. Determine the rate of entropy generation for this process.

Continuity Eq.: $\dot{m}_i = \dot{m}_e = \dot{m}$

Energy Eq.: $0 = \dot{Q}_{\text{c.v.}} + \dot{m}h_i - \dot{m}h_e - \dot{W}_{\text{c.v.}}$

Entropy Eq.: $0 = \dot{m}(s_i - s_e) + \dfrac{\dot{Q}_{\text{c.v.}}}{T_0} + \dot{S}_{\text{gen}}$

From the R-410a tables, B.4, we get

$$h_i = 280.6\ \text{kJ/kg}, \qquad s_i = 1.0272\ \text{kJ/kg K}$$

$$h_e = 307.8\ \text{kJ/kg}, \qquad s_e = 1.0140\ \text{kJ/kg K}$$

From the energy equation,

$$\dot{Q}_{\text{c.v.}} = 0.08\ \text{kg/s}\ (307.8 - 280.6)\ \text{kJ/kg} - 3.0\ \text{kW} = 2.176 - 3.0 = -0.824\ \text{kW}$$

From the entropy equation,

$$\begin{aligned}\dot{S}_{\text{gen}} &= \dot{m}(s_e - s_i) - \frac{\dot{Q}_{\text{c.v.}}}{T_0} \\ &= 0.08\ \text{kg/s}\ (1.0140 - 1.0272)\ \text{kJ/kg K} - (-0.824\ \text{kW}/303.2\ \text{K}) \\ &= -0.001\,06 + 0.002\,72 = +0.001\,66\ \text{kW/K}\end{aligned}$$

9.5 Engineering Applications; Efficiency

thermal efficiency for a heat engine cycle

$$\eta_{th} = \frac{W_{net}}{Q_H}$$

Denoting the work done in the real process *i* to e as w, and that done in the ideal, isentropic process from the same P_i, T_i to the same P_e as w_s, we define the efficiency of the turbine as

$$\eta_{turbine} = \frac{w}{w_s} = \frac{h_i - h_e}{h_i - h_{es}}$$

The compressor (or pump, in the case of a liquid) efficiency is defined as

$$\eta_{comp} = \frac{w_s}{w} = \frac{h_i - h_{es}}{h_i - h_e}$$

If an effort is made to cool a gas during compression by using a water jacket or fins, the ideal process is considered a reversible isothermal process, the work input for which is w_T, compared to the larger required work *w* for the real compressor. The efficiency of the cooled compressor is then

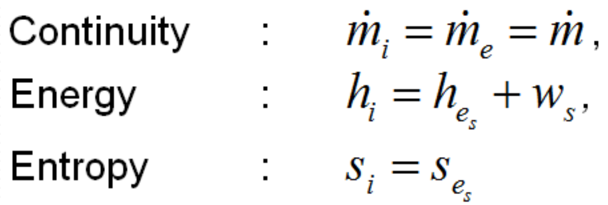
$$\eta_{cooled\ comp} = \frac{w_T}{w}$$

The nozzle efficiency is defined in terms of the corresponding kinetic energies,

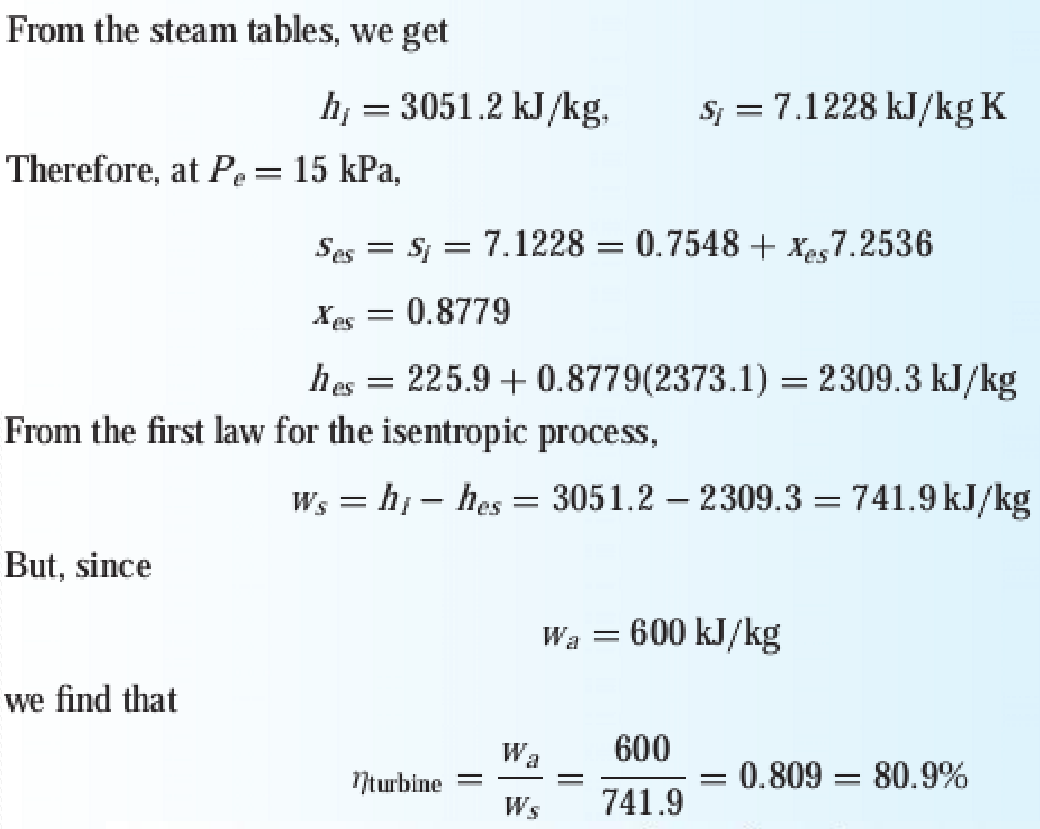
$$\eta_{nozzle} = \frac{V_e^2/2}{V_{es}^2/2}$$

EXAMPLE 9.10 A steam turbine receives steam at a pressure of 1 MPa and a temperature of 300°C. The steam leaves the turbine at a pressure of 15 kPa. The work output of the turbine is measured and is found to be 600 kJ/kg of steam flowing through the turbine. Determine the efficiency of the turbine.

$$\eta_{turbine} = \frac{w_a}{w_s}$$



Continuity	:	$\dot{m}_i = \dot{m}_e = \dot{m}$,
Energy	:	$h_i = h_{e_s} + w_s$,
Entropy	:	$s_i = s_{e_s}$



From the steam tables, we get

$$h_i = 3051.2 \text{ kJ/kg}, \qquad s_i = 7.1228 \text{ kJ/kg K}$$

Therefore, at $P_e = 15$ kPa,

$$s_{es} = s_i = 7.1228 = 0.7548 + x_{es} 7.2536$$

$$x_{es} = 0.8779$$

$$h_{es} = 225.9 + 0.8779(2373.1) = 2309.3 \text{ kJ/kg}$$

From the first law for the isentropic process,

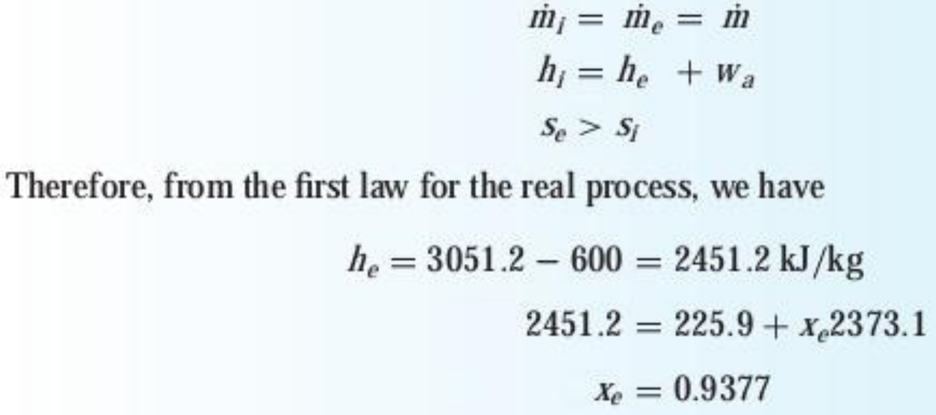
$$w_s = h_i - h_{es} = 3051.2 - 2309.3 = 741.9 \text{ kJ/kg}$$

But, since

$$w_a = 600 \text{ kJ/kg}$$

we find that

$$\eta_{\text{turbine}} = \frac{w_a}{w_s} = \frac{600}{741.9} = 0.809 = 80.9\%$$



$$\dot{m}_i = \dot{m}_e = \dot{m}$$

$$h_i = h_e + w_a$$

$$s_e > s_i$$

Therefore, from the first law for the real process, we have

$$h_e = 3051.2 - 600 = 2451.2 \text{ kJ/kg}$$

$$2451.2 = 225.9 + x_e 2373.1$$

$$x_e = 0.9377$$

EXAMPLE 9.11 Air enters a gas turbine at 1600 K and exits at 100 kPa and 830 K. The turbine efficiency is estimated to be 85 %. What is the turbine inlet pressure?

The efficiency, which is 85 % $\eta_{turbine} = \dfrac{w}{w_s}$

The first law for the real, irreversible process $h_i = h_e + w$

The first law for the ideal, isentropic process $h_i = h_{es} + w_s$

The second law $s_{es} - s_i = 0 = (s^o_{es} - s^o_i) - R\ln\dfrac{P_e}{P_i}$

From air tables, Table 2.4.3, at 1600 K, we get
$h_i = 1756.42\, kJ/kg \qquad s^o_i = 3.92522\, kJ/kg.K$

From the air tables at 830 K (the actual turbine exit temperature),
$h_e = 856.91\, kJ/kg$

Therefore, from the first law for the real process
$w = h_i - h_e = 1756.42 - 856.91 = 899.51\, kJ/kg$

Using the definition of the turbine efficiency,
$w_s = 899.51/0.85 = 1058.25\, kJ/kg$

From the first law for the isentropic process,
$h_{es} = h_i - w_s = 1756.42 - 1058.25 = 698.17\, kJ/kg$

so that, from the air tables,
$T_{es} = 683.6\, K \qquad s^o_{es} = 2.9480\, kJ/kg.K$

and the turbine inlet pressure is determined from
$$0 = 2.9480 - 3.92522 - 0.287\ln\frac{100}{P_i}$$

P_i = 3011 kPa

EXAMPLE 9.12 Air enters an automotive supercharger at 100 kPa and 300 K and is compressed to 150 kPa. The efficiency is 70 %. What is the required work input per kg of air? What is the exit temperature?

The efficiency, which is 70 %

$$\eta_{comp} = \frac{w_s}{w} = \frac{h_i - h_{es}}{h_i - h_e}$$

The first law for the real, irreversible process is

$$h_i = h_e + w \quad \Rightarrow \quad w = C_p(T_i - T_e)$$

For the ideal, isentropic process from the first law is

$$h_i = h_{es} + w_s \Rightarrow \quad w_s = C_p(T_i - T_{es})$$

and the second law $\dfrac{T_{es}}{T_i} = \left(\dfrac{P_e}{P_i}\right)^{(k-1)/k}$

Using C_p and k from Table 2.2, from the second law, we get

$$T_{es} = T_i\left(\frac{P_e}{P_i}\right)^{(k-1)/k} = 300*\left(\frac{150}{100}\right)^{(1.4-1)/1.4} = 336.9\,K$$

From the first law for the isentropic process, we have

$$w_s = C_p(T_i - T_{es}) = 1.0035*(300 - 336.9) = -37.03\,kJ/kg$$

so that, from the efficiency, the real work input is

$$w = -37.03/0.7 = -52.90\,kJ/kg$$

and from the first law for the real process, the temperature at the supercharger exit is

$$w = C_p(T_i - T_e) \quad \Rightarrow \quad T_e = T_i - w/C_p = 300 - \frac{-52.90}{1.0035}$$

T_e = 352.7 K